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This is the final technical report on the above referenced grant on the subject of robust control and robust identification. The work under the grant focused on the following topics: multiple objective robust controller synthesis, robust control analysis and synthesis, robust identification including frequency domain identification in H-infinity and model validation, implementation of gain scheduled controllers for nonlinear control, and engineering applications of the theoretical results. The report gives a summary overview of the results in these research areas. It also contains a complete list of papers which have been published under the grant.

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Robust Control and Robust Identification

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FINAL REPORT

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1 Introduction

Our research supported under this grant has focused on various problems in robust control and robust identification. We have worked on the following topics:

- Multiple Objective Robust Controller Synthesis
- Robust Control Analysis and Synthesis
- Robust Identification — Frequency Domain Identification in $\mathcal{H}_\infty$, Model Validation
- Implementation of Gain Scheduled Controllers for Nonlinear Control
- Engineering Applications

Since much of our work has already appeared in a number of journal and conference publications, and in order to keep this report concise, we will simply summarize the main contributions in these various research directions. The interested reader can find complete details, precise mathematical formulations, and proofs in the papers cited in the report. Furthermore, again motivated by concerns for brevity, we will restrict the discussion entirely to our papers supported under this grant. For a complete discussion of relevant work by other researchers, we refer the reader to our publications cited below.

2 Multiple Objective Robust Controller Synthesis

Consider the feedback system shown in Figure 1. The plant to be controlled is denoted by G, while the controller is denoted by C. The exogenous inputs are $w_0, \ldots, w_s$ (these are signals such as sensor noises, load disturbances, commands, input channels for modeling uncertainty,) The controlled or regulated outputs are $z_0, \ldots, z_s$ (these signals represent weighted tracking errors, weighted actuator inputs, output channels for modeling uncertainty). The control input vector is u while the

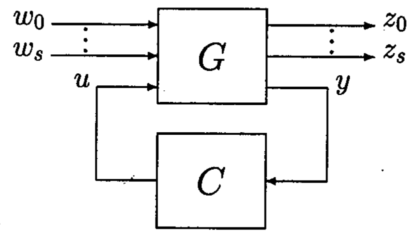

Figure 1: The synthesis framework

measured output vector is y. The input-output operator from w_h to z_h will be denoted by $T_h(C)$. We are primarily interested in performance measures of the form

$$J_h(C) := \|T_h(C)\|_{\alpha_h},$$

where α_h indicates the norm of interest. Typically, $\alpha_h = 2, \infty, \mathcal{A}$. These norms are the most commonly used system norms in robust control.

Most of our work in this area has been focused on the so called mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problems. The basic notion is to minimize the $\mathcal{H}_2$ norm of a given closed loop transfer function subject to a constraint on the $\mathcal{H}_\infty$ norm of another closed loop transfer function. The $\mathcal{H}_\infty$ norm constraint may represent, for example, a robust stability constraint while the $\mathcal{H}_2$ norm objective function may represent a performance metric. Thus, a typical problem is to minimize J_1 subject to $J_2 < 1$ with $\alpha_1 = 2$ and $\alpha_2 = \infty$.

In previous work, we had given convex optimization based approaches to a version of this mixed $\mathcal{H}_2/\mathcal{H}_\infty$ probem in which the actual $\mathcal{H}_2$ norm is replaced by an upper bound on the $\mathcal{H}_2$ norm. The upper bound depends on the $\mathcal{H}_\infty$ norm constraint. The resulting problem then becomes a tractable approximation to the "exact" mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem. This approximation by initially formulated by D. Bernstein and W. M. Haddad and has been adopted by a number of researchers. In [8], we showed via some numerical examples, that the optimal controller for the approximate mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem can yield a $\mathcal{H}_2$ performance which is actually worse than the central $\mathcal{H}_\infty$ controller for the pure $\mathcal{H}_\infty$ suboptimal controller design problem. This is quite surprising in that the $\mathcal{H}_\infty$ central controller is completely independent of the $\mathcal{H}_2$ norm objective!! What this shows is that in optimiz-

ing an upper bound on the actual objective function one may do worse than even ignoring it. The reason for this interesting behavior is that the upper bound approximation becomes quite bad as one gets close to the $\mathcal{H}_\infty$ norm constraint, as J_2 get s close to 1.

Most of the results in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control theory are restricted to the case of one (vector) input and two (vector) outputs (or the dual problem of two vector inputs and one vector outputs.) In earlier work, we had obtained some sufficient conditions for the general two vector inputs and two vector outputs case. These results were further generalized in [16, 15]. This generalization combined the best features of our eariler results is perhaps the most general result of its kind in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ theory.

In a tracking problem context, we posed the problem of robustly tracking sinusoidal reference signals in the presence of norm bounded unstructured uncertainty in [7]. A complete analytical solution was also given in this paper. The solution involves solving certain linear matrix equations and algebraic Riccati equations.

In [14], we gave a $\mathcal{H}_2/\mathcal{H}_\infty$ formulation in a filtering and estimatio n context, and obtained results on the structure of the solution. We also provided a general convex optimization based solution to this problem. This pape r also contains some promising simulation results showing robustness of the resulting filters to variations in the noise spectral properties.

3 Robust Control Analysis and Synthesis

Motivated by the widespread use of digital control, in the last few years, a number of researchers have been focusing their efforts on developing new techniques for the analysis and synthesis of robust sampled-data systems. This research direction has the promise of providing tools for the analysis and design of digital control systems and understanding tradeoffs between sampling rate, performance, and robustness.

Since frequency response occupies such a central place in the theory of control systems, we have

developed procedures for computing frequency response of sampled-data systems [23, 22]. This is not straightforward because sampled-data systems are not time-invariant. But their periodic time-varying nature makes this feasible. In order to facilitate numerical computation of the frequency response, some upper bounds on the gain of the frequency response operator were obatained in [24]. These frequency response techniques are also very useful for signal reconstruction problems which arise in signal processing as shown in [26, 27]. This may develop into a nice new research direction.

A time domain model involving differential equations with discontinuous jumps for sampled-data systems was proposed in [1]. Based on this model, a complete characterization of the $\mathcal{L}_2$ norm of a closed loop sampled-data system was obatined in terms of Riccati differntial and albebraic equations. These equations lead to state-space type computational procedures for the $\mathcal{H}_\infty$ norm in the sampled-data systems case.

A close relation between state-feedback controllers which optimize stability robustness in a gap metric setting and the classical LQR solution was given in [6]. This is of interest since it showed an additional interesting property of the LQR state-feedback controllers.

It is well known that the stability and performance robustness analysis problems for real parametric uncertainty is a very difficult problem. During this grant period, we made some progress on this problem. First, we gave an analytical procedure to compute the average case and worst-case $\mathcal{H}_2$ norm of a closed loop system in the presence of real parametric uncertainty. The results are particularly useful for one real parameter entering linearly in the systems matrices. This paper also introduced the interesting concept of average case analysis in a robsut control context.

It has been shown many researchers that the robust stability analysis against real perturbations problem is NP hard. In what may turn out to be a breakthrough direction, we took a probabilistic appraoch to this problem in [25]. We showed that there exist randomized algorithms that have only polynomial complexity. This work may start a new trend towards a more "experimental" approach to robust control theory.

One approach to deal with transient response specifications is to impose a constraint on the lo-

cation of closed loop poles. A problem of optimal $\mathcal{H}_2$ controller design subject to constraints on the closed loop pole locations was solved in [20, 19]. The solution involves solving a pair of algebraic Riccati equations, much like the classical LQG solution.

In a completely different direction, we [2] considered the problem of decentralized control and showed that under a very mild connectivity assumption, one can always stabilize a linear time-invariant plant using decetralized control using periodic feedback showing yet again the power of periodic feedback.

4 Robust Identification

Our initial work was in the area of identification in $\mathcal{H}_\infty$. The problem here is to obtain a model for the system under consideration from possibly noisy frequency response data. We are also interested in obtaining bounds on the modeling error which could be utilized for robust controller design and analysis. These error bounds depend both on the a priori information on the unknown system as well as the algorithm. We have obtained some very effective and easily computable solutions to the problem of identification in $\mathcal{H}_\infty$. These algorithms have the merit of being computationally very efficient. They involve using the fast Fourier transform (FFT) algorithm and Hankel singular value computations. We have also obtained error bounds on the modeling error.

In terms of technical results, we considered the problem of identification in $\mathcal{H}_\infty$ for nonuniformly spaced frequency response data in [4]. This problem naturally arises in continuous-time systems, and is also a natural problem for discrete-time systems. A solution procedure was also provided in [4] along with a careful analysis of error bounds and convergence rates . Unfortunately in this general case, we could not obtain exponential converegence rates although that result has just been obtained by Dr. H. Akcay.

While there has been a lot of work on obtaining bounds on model uncertainty, from a practical point of view, these bounds do not appear to be useful in the sense the assumed a priori information is difficult to obtain. As a matter of fact, while there has been some progress in this direction

in the recent literature, this remains an important issue for future research. This will remain a focus of our research. One promising avenue for this is to combine system identification with model (in)validation. The idea is to postulate a model for the system along with bounds on the uncertainty. This could come from performing system identification experiments. Then with fresh data, one could pose the question whether the data is consistent with the postulated model. This precise scenario was treated in our prize winning paper [5] where we gave an analytical solution to a time-domain model validation problem.

One of major efforts under the current grant was to develop methodologies for making the algorithms for identification in $\mathcal{H}_\infty$ applicable to real data and systems. This was the topic of Ph D dissertation of Dr. Jonathan Friedman. Much of this work has been summarized in the applications papers [10, 11]. The focus of these papers is on the development of techniques to facilitate the application of algorithms for identification in $\mathcal{H}_\infty$ to real experimental data which can be collected using commercially available equipment such as frequency analyzers. In addition to these applications, R. Rajamani at GE has also applied these algorithms, with excellent results, to modeling a high power combustor.

In a more traditional direction, we analyzed the performance of the classical least squares algorithm in the presence of worst case bounded noise. After deriving some bounds on the parameter estimation error, we showed that the least squares algorithm is robustly convergent in the sense defined in the worst-case identification literature.

The last decades have seen tremendous progress in the design methodologies for linear and nonlinear control systems. Obviously, to utilize these advanced methods, it is necessary to have suitable models for the processes to be controlled. By comparison to controller synthesis, the progress in system identification and modeling has not been as vigorous. Therefore, we feel that it is important and useful, from an applications standpoint, to focus on the process of obtaining control oriented models for nonlinear systems. Thus, a major new direction in our research in system identification is focused on nonlinear systems. This has arisen out of our work in the area of control of semiconductor manufacturing. Since first principles models are not available, we have been forced to use system identification techniques for this class of problems. In [21], we have developed prac-

tical techniques for the identification of nonlinear systems using Hammerstein models. The resulting models are in a form suitable for controller design.

5 Gain Scheduled Controllers for Nonlinear Control

Recently, we have obtained a method [12, 13] for implementation of gain scheduled controllers for nonlinear systems that has the useful property that the robustness and stability properties of the linear design are locally preserved for the closed loop nonlinear system. This technique is very easy to implement on real engineering systems. We have already used it a number of applications studies.

6 Engineering Applications

In addition to the applications work on flexible system frequency response identification, our major efforts in applying modern control methods is in the area of control of semiconductor manufacturing processes. Here we have been exploring a variety of issues including nonlinear system identification from real data, nonlinear control, [21] etc. An attractive feature of this activity is that we are able to implement and test or algorithms. This is enormously important in that the insight offered by practical implementation can not be replaced by software experiments. In addition, we are beginning to develop applications projects in the area of automotive systems control [18].

7 Ph. D. Students Supported under this Grant

We list below the names and dissertation titles of students were supported under this grant.

1. Dr. D. Shim

Thesis Title: Analysis and Synthesis of Linear Time-Invariant Systems with Positive Real Uncertainty

Graduation Date: December 1993

2. Dr. J. Friedman

   Thesis Title: Modeling, Identification, and Control of Flexible Systems

   Graduation Date: April 1996

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